OUTLINE OF THE PROPOSED TOPIC OF RESEARCH

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1 Proposed Topic of Research

Structural Transformations on Multidimensional Networks

2 Objectives of the Proposed Research

Objectives of the proposed research are:

1. To study existing structural transformations on monodimensional and multidimensional networks.
2. To analyze network properties of multidimensional networks using real and generated data sets.
3. To propose structural transformations on multidimensional networks and study their effect on the network properties.
4. To propose guidelines for design of multimodal transport networks, virtual data networks and critical infrastructure networks.

Keywords: Multidimensional Networks, Dimension Extraction, Network Science, Infrastructure Networks, Virtual Networks.

3 Background of the Proposed Research

3.1 Introduction

Network science consists of the study of network representations of physical, biological, and social phenomena leading to predictive models of these phenomena [1]. Network representations provide new insights into global structure not only of man-made networks like Internet, but also of the dynamics of epidemics, research collaboration, library usage etc. [2, 3]. Network science is an interdisciplinary area of research that combines research insights from graph theory, statistical physics, computer science, social network theory, social networks, biology, statistics etc. Network science based approach has been used to model social relations, transport networks, data networks, economy, financial markets, world wide web (WWW) etc [4].

Theoretical foundations for study of network science were laid by Erdös and Rényi through their random graph model [5]. Erdös-Rényi random graph model was the first model to explain the structure and evolution of a network. Researchers have also used percolation[1] theory to explain phase transitions[2] in random graphs [3]. Small-world model [6] and scale-free model [7] are the other two widely used network models. Researchers created models such as modified scale-free networks, power-laws to account for the differences between the existing network models and real world networks [3].

Many empirical studies of networks undertaken over the last 20 years resulted in large data sets. A few examples of these large data sets are: one-day mobile call graph consisting of 50 million nodes and 170 million links, text and call graph of a Belgian mobile phone company with 2.5 million nodes and 810 million edges over a period of six months [4]. Study of these large networks – called as complex networks – requires use of network sampling techniques [8]. Such large graphs also lend themselves to statistical analysis of network properties. Historically, complex networks have been analyzed for network structure, node degree distribution, centrality, presence of hubs, evolution, giant component etc [2]. These networks in addition to being complex, are also evolving. Properties of evolving complex networks include preferential attachment, robustness of a network w.r.t. node and link failures, shrinking diameters and other forms of densification governed by power laws etc.

3.1.1 Multidimensional networks

Most real world networks have nodes with multiple connections between them. Often multiple connections indicate different kinds of interactions and require labeling of edges to distinguish

[1]Percolation is the phenomenon of a fluid or substance passing through a porous substrate; percolation is used in network science to explain epidemics, propagation of ideas etc.

[2]Phase transition is the transformation of a thermodynamic system from one state to another; phase transition is used in network science to explain the formation of giant component in the network.

between different kinds of interactions [9]. These different kinds of interactions can be studied by using multi-layered graphs, each layer corresponding to one kind of interaction. Researchers have given names such as multidimensional networks [9], multi-layer networks [10], layered social network [11], multirelational networks [12], multimodal networks [13] to describe the networks that have nodes with multiple connections between them. Mathematical abstraction *multigraph* is not be confused with a *multidimensional network*. In multigraphs, two vertices may be connected with multiple edges of same kind of interaction; On the other hand, each dimension of a multidimensional network represents a different kind of interaction [12].

Most widely used models and properties of networks classify all links of a network as representing same kind of interaction; theory developed for such networks of single link type is applicable only for monodimensional networks. Theoretical foundation required for studying multidimensional networks is still evolving [9]. Properties of interest like centrality and evolution, well understood in the context of monodimensional networks, need to be redefined in the context of multiple dimensions [10].

3.1.2 Structural Transformations on Multidimensional Networks

The existence of relationships between nodes, i.e. inter-dimensional relationships across multiple dimensions in a multidimensional network necessitates an approach to model these relationships using a graph. The inter-dimensional relationship between nodes has either been described as repetition of nodes in more than one dimension [14], an undirected and unweighted graph [15] or random graph [16]. In general, inter-dimensional relationships between nodes can be modeled as directed and weighted graph. In the context of this proposal, structural transformations on multidimensional networks imply transformations on graphs that represent inter-dimensional relationships. *Aggregation, dimension extraction* and *design of robust multidimensional network* are three broad topics of interest under structural transformations on multidimensional networks. Aggregation in multidimensional networks entails the process of combining multiple dimensions into an equivalent monodimensional network [17]. Dimension extraction involves creation of useful logical dimensions from existing dimensions of a network [15]. The design of robust multidimensional network involves connecting two or more monodimensional networks in a way that satisfies design goals [18]. Structural transformations on multidimensional network is a dynamic property that requires further study [17].

Study of structural transformations on multidimensional networks has been found to be useful in the areas of transport networks [14], virtual networks [19] and critical infrastructure networks [18]. Logical traffic network has been used to represent railway traffic [14]. Virtual networks is an emerging area in data networks that is used by the network service providers to provision virtual (logical) network on top of diverse physical networks [19]. Design of multimodal transport networks - combined design of rail, road and air transportation networks - has been undertaken using multidimensional networks approach [13]. Multidimensional network design approach has been used to reduce probability of cascade failure between power grid and communication system [18].

The effect of node arrival or departure in monodimensional network can be understood as inter-dimensional link formation or deletion in a multidimensional network. For example, creation of a metro railway station at an airport is akin to inter-dimensional link formation between metro railway and airport networks. Effect of inter-dimensional link formation and deletion in a multidimensional network is not clearly understood. Tendency of monodimensional networks to connect with each other increases the size of effective multidimensional network.

3.2 Literature Survey

Literature survey contains four sections.

- First section describes structure, properties and dynamics of monodimensional networks.
- Second section explains the importance of multidimensional networks.
- Third section provides an overview of mathematical models used to represent multidimensional networks. A review of applications that use these mathematical models is also provided in third section.
- Fourth section provides a review of existing structural transformations on multidimensional networks.

3.2.1 Structure, Properties and Dynamics of Monodimensional Networks

Significant research effort in the field of network science has been directed towards determining structure and properties of networks observable among the real world networks like Internet, WWW, Peer-to-Peer network etc. Erös and Rényi's random graph model was the first mathematically rigorous approach to understand the structure of a network [5]. Next significant structural model of the network is *small world network model* proposed by Watts and Strogatz [6]. Milgram, Pool, and Kochen also studied the small world model [3]. Networks are said to show *small world effect* if mean length of the shortest paths (mean geodesic distance) between vertex pairs scales logarithmically or slower with network size for a fixed mean degree. Almost at the same time, two significant and related discoveries were made on structure of networks; One is the networks with structures described by power laws and the other is a scale-free network [7]. Some researchers also refer to networks described by power laws as scale-free networks. The random, small world, and scale-free network models are still considered as fundamental network models. Variations of fundamental models have also been studied by researchers.

Study of interactions and their outcome is the central theme of network science. Properties of a network determine the nature and quality of interactions among nodes of a network. Early researchers used network models to determine network properties such as average path length, clustering coefficient, degree distribution, giant component etc [2, 3]. A more representative definition of network properties cosiders the context of application. Researchers use *centrality* as a notion that describes a defining property of a network. Most widely used centrality properties are: degree centrality, betweenness centrality, and Eigen vector centrality [2].

Networks of the real world are dynamic – they evolve with time. This evolution may be understood as addition or deletion of nodes and / or links. Network can also become dynamic by a change in link attributes like directionality, weight w.r.t. time. These dynamic networks are also referred to as *temporal networks, evolving networks* etc. A review of evolving networks by Dorogovtsev et al. [20] emphasizes on node addition aspect of evolving networks. Network growth models like preferential attachment and weight-driven growth are attempts at explaining the growth of a network. Recently, attempts have been made to redefine centrality measures for dynamic networks [9].

3.2.2 Importance of Multidimensional Networks

Human beings and the social / technological systems are complex and interact at many levels [12]. These different interactions give rise to multiple relationships between participants. In order to analyze properties of these systems with multiple relationships, researchers have been using the notion of multidimensional networks. These multidimensional networks may also have inter-dimensional edges that change the structure and dynamics of networks [21]. Application data sets like flickr, bibliographic database (DBLP) etc can be modeled appropriately only by a multidimensional network [22]. Multidimensional network can also be constructed by using two or more monodimensional, real-world networks [16]. There have even been attempts to model business organizations as multidimensional networks [12]. In the past, each dimension of a multidimensional network has been studied separately [4]. However, this study of interacting monodimensional networks as independent networks fail to account for many of the properties of multidimensional networks considered as a whole [12]. Multimodal transport networks [13], virtual data networks [19, 23], and critical infrastructure networks [24, 25] are few areas where multidimensional network-based analytical approach is necessary.

Interacting networks which are to be modeled as multidimensional networks usually exhibit a systems phenomenon known as *emergence*. Emergence refers to occurence of a phenomenon at system level; the origin of this phenomenon cannot be directly attributed to sub-components of the system. Standard node properties like *degree* and *hub* require redefinition in the context of multidimensional networks [9]. Emergence of multidimensional hubs are attributable only to the multidimensional nature of the network [22].

3.2.3 Representation and Study of Multidimensional Networks

Network science deals with representation of real-world phenomenon by using network models. Construction of a good model requires creation of proper network representation to study the phenomenon of interest [2]. Thus proper representation of a multidimensional network is a requisite first step in order to study multidimensional networks [10].

A simple approach for representing multidimensional networks is to represent network of each dimension as a separate network. In fact most research results in network science apply to a single dimension of a multidimensional network. Szell et al. [12] apply graph union operation on all dimensions of a multidimensional network to create an *envelope network*. It has been found by authors that such a reduced network representation does not yield useful results. Another approach is to form a *partial envelope network* by performing a combinatorial sum type of aggregation on dimensions.

Random network generation model can be used to create and then study multidimensional networks [16]. In this approach, network of each dimension is generated using standard network models such as small-world, scale-free or random networks. Nodes of these monodimensional networks are mapped across dimensions to create a multidimensional network; mapping function can either be proximity-based or probabilistic. The resultant multidimensional network is studied for properties of interest.

A two-layer approach has been used by Kurant and Thiran [14] to represent multidimensional aspect of transport networks. Authors use the term *physical graph* to represent independent physical railway network; the term *logical graph* has been used to represent dependent logical railway traffic network. Logical graph is derived from physical graph and both are stored separately. Authors also provide an algorithm to extract logical graph from physical graph. This two-layer network model correctly represents multidimensional networks which have a single independent dimension (physical graph), i.e. all other dimensions are derived dimensions (logical graphs).

A similar approach to store multidimensional data network as a three layer network is proposed by Pacharintanakul and Tipper [15]. Wavelength division multiplexing (WDM) physical links of a data network are represented using an equivalent *physical graph*. A *logical IP network* formed on top of WDM physical links is represented using a logical graph whose topology is a derivative of the physical graph. A higher logical graph called *overlay* is derived using logical *IP* network. Overlay graph is appropriate for users of data networks while logical IP network and physical network are appropriate for Internet Service Providers (ISPs). This approach explains the use of physical graph - logical graph abstractions in the context of data networks. Extraction of dependent logical dimensions from independent dimensions is referred to as *dimension extraction*.

Berlingerio et al. [26] model complex relationships between nodes of multidimensional networks as edge labeled multigraphs. As per this notation, each edge of a graph has a label associated with it; the label itself comes from a set of labels. Each edge of the multigraph may also have a weight

associated with the edge. Edge labeled multigraphs provide a compact graph representation for multidimensional networks. This representation has been used to model transportation networks, social networks, co-authorship networks and utility networks.
Magnani et al. [10] propose a *pillar model* of representation for the multidimensional networks. In pillar model, all the dimensions of the network are represented as individual networks. Each user metaphorically represents a pillar which gets mapped to a vertex or a set of vertices on a layer. The vertices of one dimension are mapped to vertices of another dimension using one-to-one and one-to-many mapping. The mapping of vertices across dimensions is indicated through identity mapping (IM) matrix. Pillar model has been used to explain the multiple identities of a person on online social networks such as *facebook, twitter, youtube, friendfeed* etc.
Kazienko et al. [17] propose a three dimensional model for multidimensional network with layers, time window and groups as the three dimensions. Layers represent different types of relationships between humans; time windows correspond to snapshot of network at a particular time and represents the dynamic nature of network; social groups correspond to sets of similar humans. A specific combination of a layer, a time period and a group in the layer is called a *view*. The idea of combining one or more views in a multidimensional graph is referred to as *aggregation by dimension*.
Gao et al. [21] use percolation and phase transition concepts from statistical physics to analyze robustness of a multidimensional network; Here, multidimensional networks are modeled as *network of networks*. A node in one dimension is assumed to have undirected edge to a single node in another dimension. Effect of failure of one node on the network of networks is studied.
As reviewed in this section, researchers are using application-specific network representations for multidimensional networks. A unified approach to represent multidimensional networks is an active area of study.

3.2.4 Structural Transformations on Multidimensional Networks

Aggregation [16, 17], *dimension extraction* [19, 23] and *design of robust multidimensional network* [18, 24] are three broad topics that come under structural transformations on multidimensional networks. A recent approach to aggregation is a study of cooperation in transport networks done by Gu et al. [16]. Authors study the possibility of cooperation in two layered networks. Both layers have same vertex set; Network in each layer (dimension) is generated using Barabási-Albert model, Erdös-Rényi model or Newman-Watts model. A cross-layer link may exist between the same node in two layers; such a link is referred to as *crossing link*. The formation of crossing links between two layers lead to cooperation between two layers. Development of cooperation between two layers, defined as *cooperation strength* has been studied as a function of intra-layer and cross-layer link costs. Authors retained the non-zero cost of crossing links in their study.
Kazienko et al. [17] represent cooperation as one form of aggregation. But their research considers

cross-layer link costs to be zero. Authors look at possible ways to operate on multiple views generated from their multidimensional network representation. Aggregation operators have been suggested as possible means to represent multiple views using an equivalent aggregated view. Sum of relation strengths, mean of relation strengths, weighted sum of relation strengths and weighted mean of relation strengths are suggested as potential aggregation operations that can be performed on multiple views.

Dimension extraction is another aspect of structural transformations on multidimensional networks. Zhang et al. [23] summarize the problem of dimension extraction in traffic engineering of data networks. Network operators frequently provision network resources based on demand. Resource provisioning is done by creating (extracting) a logical layer from physical layer; here logical layer is a subset of physical layer and represents a fraction of network resources of physical layer. The extraction of logical layer from physical layer can be formulated as an optimization problem. Authors outline algorithms for extraction of logical layers using greedy algorithm and simulated annealing algorithm.

Kurant and Thiran [14] use logical graph to represent possible railway traffic load on physical graph of railway network. They create logical graph based on physical graph, but the edge weights of logical graph have no relation to edge weights of physical graph. The general idea of dimension extraction is also used by network researchers to create virtual private networks (VPNs), programmable networks and overlay networks on top of diverse infrastructure data networks. VPNs, programmable and overlay networks are different facets of network virtualization. Chowdhury and Boutaba [19] provide an overview of network virtualization.

Van Nes [13] proposes a hierarchical approach for design of multimodal transport networks. Here multimodal transport implies using two or more modes for making a trip, i.e. different vehicle modes (air, rail,bus etc) or functionally different service modes (private, public etc). Research by Van Nes looks into design of geographically non-overlapping, independent but interacting networks. A few nodes where the independent networks interact are called as *transfer nodes*. Design of efficient multimodal transport network requires proper selection of transfer nodes to satisfy all stakeholders.

Another approach to creating a desirable merged urban infrastructure network has been proposed by Ouyang and Osorio [24]. Urban infrastructure systems like gas transmission system and power grid are represented as single dimensions of a multidimensional network. Physical connections across infrastructure systems are referred to as *interface topology*. Authors try to develop design strategies for creating interface topologies for the urban infrastructure networks. Overall objective is to tune the design strategy for desired infrastructure topological properties, geographical features and reliability information. The chosen strategy is judged by reduction in the possibility of cascade failures.

Buldyrev et al. [27] study domino failure scenario in interdependent networks. As part of study, authors create two interdependent networks and randomly connect the networks. Failure of a node in one network leads to failure of all adjacent nodes of failed node in second network. This failure scenario is categorized as *cascade of failures* and such interdependent networks are called as *coupled networks*. Emphasis however is on the robustness of a network to failure of nodes in another coupled network.

There is a renewed emphasis on design of robust coupled networks among researchers. Schneider et al. [18] try to identify the minimum set of nodes that need to be decoupled to obtain a desirable level of robustness. Decoupling makes a node immune to failure of a connected node in the corresponding coupled network. Decoupled nodes are referred to as *autonomous nodes*; identification of autonomous nodes themselves is done on the basis of betweenness and degree.

As reviewed in this section, aggregation, dimension extraction and design of robust multidimensional network are three broad topics that come under structural transformations on multidimensional networks. Researchers have studied aggregation, dimension extraction and multidimensional network design separately. These attempts are far from complete in the face of continuous growth in complexity and variety of multidimensional networks. There is an overwhelming need to study these three topics together for a better understanding of multidimensional networks.

3.3 Gaps in Existing Research

Key open research issues include the following:

- Modeling and analysis of multidimensional networks has been done using unweighted and undirected networks [18, 26]. Schneider et al. [18] suggest using weighted network as one possible way to extend their work on robustness of coupled networks. The multidimensional model proposed by Kazienko et al. [17] does not take direction and weight of an edge into consideration. But direction and weight of an edge can have important consequences in social networks such as influence networks. Use of directed and weighted network representations for multidimensional networks makes these network models widely applicable.
- *One-to-one* and *one-to-many* IM between dimensions of a multidimensional network have been studied by Magnani and Rossi [10]. Authors hint at *many-to-many* IM, but did not study the possibility. Many-to-many IM models the scenario of IM for an organization / group across dimensions. Used this way, many-to-many IM model helps explain the patterns of inter-group communication that spans multiple online social networks (dimensions). Use of *many-to-many* IM between dimensions of a multidimensional network requires further study.
- Algorithms for extraction of logical network from physical network have been written for monodimensional network [14, 15]. Authors of both the papers point out that creation of a suitable logical network enables error and attack tolerance analysis on logical network. Also the centrality measures of logical network are likely to be different from those of pysical

networks [14]. The scale of networks, often with more than 2000 vertices, eliminates use of manual procedures for extraction of logical network. Hence there is a need to extend existing dimension extraction algorithms for multidimensional networks.

- Two possible structural transformations on a network – aggregation [17] and dimension extraction [14], have been studied independently. To understand real world networks better, there is a need to study both aggregation and dimension extraction together on a multidimensional network.

4 Methodology

Phase 1: Literature survey and analysis of existing data sets.

Literature survey shall be undertaken to review published research results on monodimensional and multidimensional networks. A detailed survey of available data sets for monodimensional and multidimensional networks shall also be undertaken. Data sets shall be used for verification of theoretical concepts of networks and for learning purposes.

Phase 2: Study of existing structural transformations on multidimensional networks.

The published work on structural transformations on monodimensional and multidimensional networks shall be reviewed. Efforts shall also be directed towards identifying suitable multidimensional network data sets for performing aggregation and dimension extraction operations.

Phase 3: Data collection / generation for multidimensional networks.

Suitable real world applications shall be identified as sources for generation of data for multidimensional networks. New algorithms shall either be designed or existing algorithms extended for generation of suitable data on multidimensional networks.

Phase 4: Structural transformations on multidimensional networks.

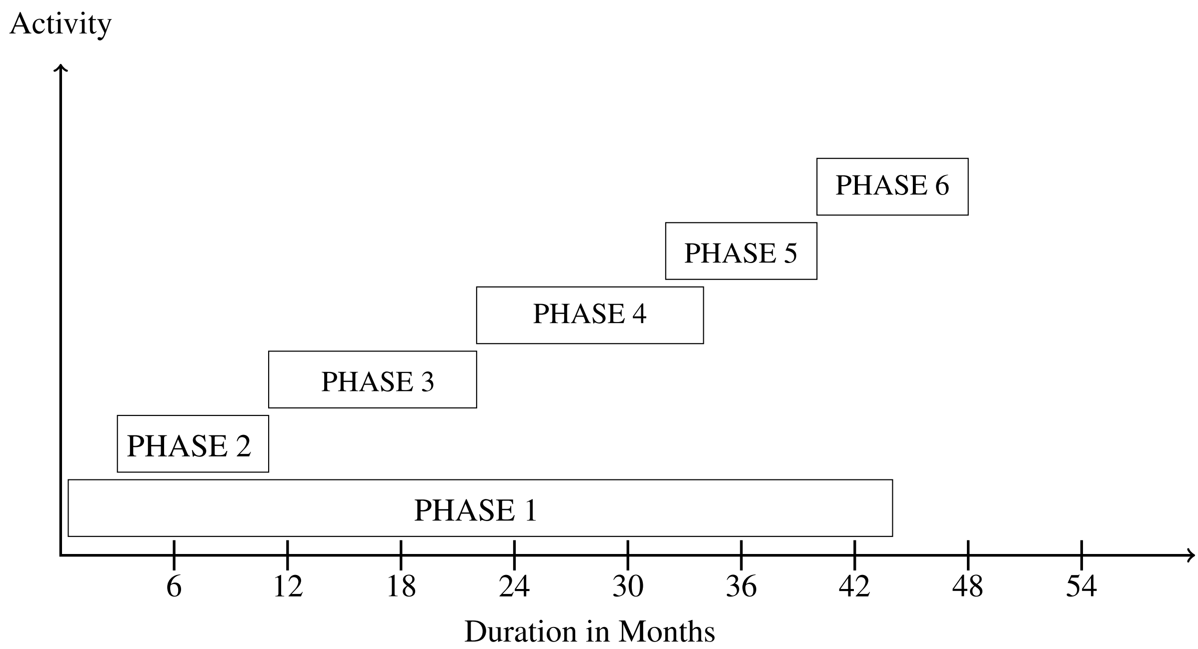
In this phase, new structural transformations on multidimensional networks shall be developed. Data sets generated in *phase 3* shall be used to validate the proposed structural transformations developed on multidimensional networks. Efforts shall also be made to identify potential applications of research results in the domains of multimodal travel networks, virtual data networks and critical infrastructure networks.

Phase 5: Design of multidimensional networks.

Multidimensional network data shall be collected from one or more of the chosen application domains. The theoretical insights to be developed during *phase 4* shall be applied to the chosen application.

Phase 6: Thesis writing.
The final phase involves documentation of research findings and submission of research findings to experts in the form of thesis document.

Plan of Work Chart



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